SPEED MODELS FOR HIGHWAY CONSISTENCY ANALYSIS. A COLOMBIAN CASE STUDY

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ABSTRACT

Most evaluations of highway geometric design consistency are based on the analysis of vehicles' speed. This paper reviews the main models of operating speed prediction on two-lane rural highways and presents a speed study carried out in Colombia. In the first phase of the research the validity for Colombia of speed models developed by researchers of other countries has been analyzed. Among the models evaluated are the one used by the Interactive Highway Safety Design Model (IHSDM). IHSDM is a computer tool developed by the Federal Highway Administration (FHWA) of the United States of America. The difficulties of their application to Colombian highways stand out, especially in mountain highways. Consequently, a speed study has been carried out with the purpose of obtaining models adapted to local conditions. Several speed models, applicable to different cases of horizontal and vertical curves, have been obtained.

Key words: highway design, consistency, speed model, IHSDM

INTRODUCTION

Design consistency refers to the condition in which roadway geometry does not violate driver expectations. Until now, the most used methods to evaluate the consistency have been based on operating speed profile analysis. These methods estimate the operating speed at which a characteristic vehicle can circulate throughout the segment in study and establishing comparisons with defined criteria (1, 2, 3, 4, 5, 6, 7, 8). When the design corresponds to which the driver hopes finding, the route is consistent. This diminishes the driver possibility of making errors and carrying out uncertain maneuvers.

The procedure of consistency analysis of the IHSDM (Interactive Highway Safety Design Model), developed by the FHWA (7) and based on speed profile analysis, stands out by the rigor of the studies on which it is based and by the simplicity of its practical application. IHSDM uses speed models calibrated in the United States of America. However, the conditions of acceleration and deceleration and, in general, the design parameters of North American highways differ from the ones used in Colombia. Another disadvantage that presents using IHSDM for consistency analysis in this country is related to the minimum design speed. IHSDM considers a minimum design speed of 60 km/h and this value is higher than the value reached in many Colombian mountain highways. In the first phase of the research, the validity for Colombia of speed models developed by researchers of other countries has been analyzed, among them, the ones used by the IHSDM. The most of non Colombian speed models are not valid for Colombian highways. Consequently, a speed study has been developed with the purpose of obtaining models adapted to local conditions.

In the first part of this paper, the main speed models are summarized. Next, the application of non Colombian speed models is evaluated. Finally, the models of operating speed prediction developed for Colombian highways are presented.

BACKGROUND

The first operating speed model was determined in the beginning of the second half of the 20^{th} century (9). Since then, around fifty models based on speed measurements made in different countries have been developed. Most authors propose determining the operating speed using the geometric characteristics of the horizontal alignment. Most models use a single variable, which is usually the radius (r) of circular curves or the degree of curvature (DC) (9, 10, 11, 13, 14, 15, 16, 17, 18, 19). In many cases these models are linear. Among them, the models of Lamm and Choueiri (1987 and 1990) (11, 12) stands out because of the high number of locations (261 curves in New York) where speed measurements took place (Table 2). Models using variables from the horizontal alignment have the advantage that the data needed for using them are easily available from cartography, ortophotos, or satellite imagery. In the model of Kanellaidis et al. (13) only one variable was also used, curvature radius, but the model was not linear because the operating speed in curve depends on the square root of the curvature radius. This work was carried out in Greece using a sample of 58 curves (Table 2). Islam and Seneviratne (14) developed models of speed at the beginning (BP), in the middle (MP) or at the end (EP) of curves using as independent variable the degree of curvature; a sample of 8 curves located in north-eastern Utah was analyzed (Table 2). In

Canada, Morrall and Talarico (15) proposed an exponential model for the speed as a function of the degree of curvature (Table 2). It is based on a sample of 9 curves.

Some authors propose models with only one variable different from the radius of circular curves or the degree of curvature. Lamm and Choueiri (11), McLean (20), SETRA (21), and Nie and Hassan (22) use the curvature change rate (CCR) (absolute angular change in horizontal direction per unit of distance). Gibreel et al. (23) determined models in Canada in three dimensional alignments, as well as a speed prediction model considering only the horizontal alignment based on the deflection angle (Ω) (Table 2). The sample used for the horizontal speed prediction model had 32 curves.

Other authors propose models with several variables based on the geometric characteristics of the horizontal alignment. Due to the high number of locations where speed measurements were made (138 curves from different regions of the United States), the model devised by Krammes et al. (5) stands out. In this model, in addition to curve radius, length of the horizontal curve and deflection angle have also a significant effect on operating speed (Table 2). It is important to note that the deflection angle is proportional to the quotient between curve length and curve radius, the other two independent variables. In a later study (with the same sample), Ottesen and Krammes (24) proposed a new model formulation which uses the degree of curvature and the length of the circular curve (Table 2). Andueza (25), based on speed measurements made in Venezuela, developed speed models for rural highways including as independent variables curvature radii in consecutive curves, length of the approach tangent and sight distance.

There are models in which, in addition to the geometric characteristics of the alignment, the effect due to cross-section has been taken into account (11, 21, 26). Lamm and Choueiri (11) propose an extended model with degree of curvature, lane width, shoulder width, and annual average daily traffic (AADT) as independent variables. They decided to drop lane width, shoulder width, and AADT from the model because they only explained about 5.5 percent of the variation in the estimated 85th percentile speed. The effect of lane width in speed prediction has been taken into account in a French model developed by SETRA (21). Voigt (26) extended the speed equation proposed by Krammes et al. (5), including superelevation (e) (Table 2). Also, in this model it is important to note the collinearity among the independent variables, i.e., radius and superelevation are highly correlated.

There are models in which, in addition to horizontal alignment, vertical alignment has been also considered. Fitzpatrick et al. (27) studied 176 places located in highways of six regions of the United States with different combinations of horizontal and vertical alignments and propose a model for speed prediction that considers both horizontal and vertical alignment. The applicable equation depends on the characteristics of the horizontal and vertical alignment (Table 1); in the most cases, it uses only the horizontal curvature radius as the independent variable, although one of the equations depends on K (rate of vertical curvature in m/%). This speed model has been implemented in the Design Consistency Module of the IHSDM (7). Bella (28) proposes a curve speed model based on 10 measurements made in a driving simulator. V_{85} depends on the CCR value of the curve and on the vertical grade.

TABLE 1. Fitzpatrick et al. speed models (27)

Combination of horizontal and vertical alignments	Model
Horizontal curve; -9 % ≤ Grade < -4 %	V_{85} = 102.10 - 3077.13 / r
Horizontal curve; -4 % ≤ Grade < 0 %	V_{85} = 105.98 - 3709.90 / r
Horizontal curve; 0 % ≤ Grade < 4 %	V_{85} = 104.82 - 3574.51 / r
Horizontal curve; 4 % ≤ Grade < 9 %	V_{85} = 96.61 - 2752.19 / r
Horizontal curve combined with sag vertical curve	V_{85} = 105.32 - 3438.19 / r
Horizontal curve combined with Non limited sight-distance crest vertical curve	V_{85} = lowest value predicted for curves with a grade between -9% and 0% (downgrade) or between 0% and 9% (upgrade)
Horizontal curve combined with limited sight-distance vertical curve (K ≤ 43 m / %)	V_{85} = 103.24 - 3576.51 / r
Sag vertical curve on horizontal tangent	V_{85} = assumed desired speed
Vertical crest curve with non-limited sight distance (K > 43 m/%) on horizontal tangent	V_{85} = assumed desired speed
Vertical crest curve with limited sight distance (K ≤ 43 m / %) on horizontal tangent	V_{85} = 105.08 - 149.69 / K

There are also curve speed prediction models that, in addition to geometric characteristics, take into consideration other variables. McFadden and Elefteriadou (18), McLean (29), Ottesen and Krammes (24) and Cardoso et al. (30) take into account the speed in the approach tangent. Cardoso used a sample of 50 curves over 4 European countries (France, Finland, Greece and Portugal). He adjusted models for each of these countries and a whole model in which there is a constant (c) has different values for each of the countries (Table 2). McFadden and Elefteriadou (18) established two speed models, one model based on degree of curvature (DC), an the other model based also on length of the circular curve (Lc), deflection angle (Ω) and speed in the approach tangent (V_{AT}) (Table 2). They used speed data measured in 78 curves in the United States. As it has been already explained, Lamm and Choueiri (11) consider, in the extended model, the traffic (annual average daily traffic -AADT-) as one of the variables. Some authors propose that speed on curves depend on the "environmental" speed (31, 32). The "environmental" speed is defined as the maximum V_{85} value in long tangents or large radius curves placed in homogeneous sections of highways. As it has been already commented, Andueza (25) proposes speed models that include among the independent variables the sight distance.

On the other hand, Lamm and Choueiri (11) suggest a speed of 94.7 km/h on independent tangents (length of tangent > 200 m), while Krammes et al. (5) 97.9 km/h, Hassan et al. (6) 102.0 km/h, and Misaghi and Hassan (17) 103.0 km/h. McLean (33) proposes the use of desired speed, defined as "the speed at which drivers choose to travel under free-flow conditions when they are not constrained by alignment features". The IHSDM allows the user to choose the desired speed on tangents.

VALIDATION OF NON COLOMBIAN SPEED MODELS FOR COLOMBIAN HIGHWAYS

In a first phase of the research, several non Colombian speed models were studied to evaluate their applicability to Colombian highways. Among them, the model used by the design consistency modulus of the IHSDM and other models developed by foreign researchers (Table 2) were analyzed. As already mentioned, the model used by the IHSDM is the one developed by Fitzpatrick et al. (27) (Table 1).

TABLE 2. Non Colombian speed models studied

AUTHOR (S)	MODEL	YEAR	COUNTRY
Lamm et al.	$V_{85} = 95.78 - 0.076\ CCR$ $V_{85} = 96.152 - 0.302\ DC$	1987	USA
Lamm et al.	$V_{85} = 94.398 - 3188.656 / r$	1990	USA
Kanellaidis et al.	$V_{85} = 129.88 - 623.1 / (\sqrt{r})$	1990	Greece
Morrall and Talarico	$V_{85} = \exp(4.561 - 0.0058\ DC)$	1994	Canada
Islam and Seneviratne	$V_{85\ BP} = 95.41 - 0.45\ DC - 0.001\ DC^2$ $V_{85\ EP} = 103.03 - 0.73\ DC - 0.003\ DC^2$ $V_{85\ MP} = 96.11 - 0.32\ DC$	1994	USA
Krammes et al.	$V_{85} = 102.44 - 2471.81 / r + 0.012\ Lc - 0.10\ \Omega$	1995	USA
Voigt	$V_{85} = 99.61 - 2951.37/r + 0.014Lc - 0.13\Omega - 71.82e$	1996	USA
McFadden and Elefteriadou	$V_{85} = 103.66 - 1.95\ DC$ $V_{85} = 41.62 - 1.29DC + 0.0049Lc - 0.12\Omega + 0.95V_{AT}$	1997	USA
Cardoso et al.	$V_{85} = 35.086 - 289.999 / \sqrt{r} + 0.759\ V_{AT}$	1998	Several
Fitzpatrick et al.	See Table 1	1999	USA
Ottesen and Krames	$V_{85} = 103.66 - 1.95\ DC$ $V_{85} = 102.44 - 1.57\ DC + 0.012Lc - 0.01DC\ Lc$ $V_{85} = 41.62 - 1.29\ DC + 0.0049\ Lc - 0.12\ DC\ Lc + 0.95V_{AT}$	2000	USA
Gibreel et al.	$V_{85} = 102.2 - 0.10\ \Omega$	2001	USA
Castro et al.	$V_{85} = 120.16 – 5596.72 / r$	2008	Spain

To do this validation speeds on Colombian roads were measured and compared with the values estimated by the models. 15 Roads and 71 sections in the South-West of the country, located in the different types of terrain considered in Colombia for highway design were selected: level, rolling, mountainous and steep.

In order to isolate the results from factors not related to highway geometry, sections should meet the following criteria:

- Two lane highways.
- Out of urban zones.
- Without crossings, bridges or tunnels.
- Good pavement conditions.
- Without speed reduction devices.

In every section, sites that met one of the ten different combinations of horizontal and vertical alignment studied by Fitzpatrick et al. (27) were located. In total, speeds have been measured in more than 200 sites. Free flow car speeds were measured with radar. Speeds of at least 40 cars were measured in each site.

In horizontal curves, speeds were measured 200 m before, in the beginning, in the middle and at the end of the curve. In vertical curves, speeds were measured in the midpoint of the previous tangent, in the minimum sight distance point and in the midpoint of the vertical curve. In horizontal curves combined with vertical ones, speeds were measured in the midpoint of the horizontal curve, in the midpoint of the vertical curve and in the midpoint of the previous spiral. In every point, the 85th percentile speed (operation speed) was determined.

For comparison among measured speeds and the estimated ones by non Colombian models, mean square error (MSE), mean absolute error (MAE), and mean absolute percentage error (MAPE) were used. Likewise, Chi-square tests were performed to verify whether data measured in Colombia conform to previously described equations. Also, the difference between the values measured and estimated was analyzed through box plots.

As an example, Table 3 shows the results of this validation tests for two speed models of Fitzpatrick et al. (27): the model for horizontal curve combined with sag vertical curve (Eq. 1) and the one for horizontal curve combined with limited sight-distance vertical curve (Eq. 2):

$$V_{85} = 105.32 - 3438.19 / r \qquad (1)$$

$$V_{85} = 103.24 - 3576.51 / r \qquad (2)$$

As could be seen in Table 3, the Chi-square calculated values are greater than the critical value. This means that there is a significant difference between the measured values and the estimated ones by these equations from Fitzpatrick et al. (27).

TABLE 3. Validation tests for two Fitzpatrick Equations

Parameter		**Equation 1**	**Equation 2**
Number of sites		15	15
Radius (m)	Range	33.6 – 892	59 – 885.26
	Mean	206.59	257.51
	Standard Deviation	232.16	259.12
Length (m) of sag curves	Range	10 – 400	N/A
	Mean	122.67	N/A
	Standard Deviation	96.40	N/A
Length (m) of crest curves with sight distance limited	Range	N/A	30 – 240
	Mean	N/A	85.33
	Standard Deviation	N/A	62.09
V_{85} (km/h)	Range	46.25 – 97	55 – 91.5
	Mean	74.79	75.65
	Standard Deviation	15.39	13.16
Mean Square Error		216.84	212.6
Mean Absolute Error		9.65	12.11
Mean Absolute Percentage Error		10.65	16.13
Calculated χ^2		64.76	41.60
Critical χ^2		25.00	25.00

In summary, the results of the comparison among measured speeds and the estimated ones by non Colombian models indicate that, except in some cases, applying in Colombia speed equations developed by researchers in other countries is not recommended, since the estimated values show significant differences with those measured in the highways. In some cases, speeds estimated are negative. This indicates that the design parameters of these sections are out of the range for which the speed models were developed.

The models that passed the validation process are:

- In horizontal curves on grades between -4% and 0% the model of Fitzpatrick et al. (27) is valid:

$$V_{85} = 105.98 - 3709.90 / r \quad (3)$$

- In horizontal curves on grades between 0 and 4% the 1990 model of Lamm et al. (12) is valid:

$$V_{85} = 94.398 - 3188.656 / r \quad (4)$$

- In tangents on vertical crest curve, without limiting sight distance the model of Fitzpatrick et al. (27) is valid:

$$V_{85} = 105.08 - 149.69 / K \quad (5)$$

SPEED MODELS PROPOSED

As a result of the non validity of most not Colombian models for Colombian roads, the task of developing speed models based on measurements made in Colombia was undertaken. To obtain operating speed models from data measured in Colombian highways three hypothesis were considered:

- Curvature effects are linear and may be added.
- There is interaction between horizontal and vertical curvature effects, since the effect of one depends on the other, and vice versa.
- The effect of one curvature prevails over the other, therefore two equations could be generated, but only the equation that corresponds with the prevailing effect is used.

Table 4 shows the independent variables that have been considered to develop operating speed models:
r = horizontal curve radius (m)
r_{n-1} = radius of the previous horizontal curve (m)
Lc = length of the horizontal curve (m)
DC = degree of curvature (º)
Ω = deflection angle (º)
K = rate of vertical curvature (m/%)
Lv = length of vertical curve (m)
W_L = lane width
e = superelevation

TABLE 4. Colombian speed models. Variables considered

Geometric combination	Variables considered
Horizontal curve	r, r_{n-1}, 1/Ω, DC
Vertical curve (sag or crest)	K, Lv
Horizontal and vertical curve combined	r, Lc, K, Lv
Cross-section	W_L, e

The coefficient of determination (R^2), the mean square (MSE), and the adjusted coefficient of determination (Ra^2) were taken into account for fitting the models. The variance inflation factor (VIF) was used to detect multicollinearity. This index increases when collinearity increases. If the VIF is greater than 10, there are severe collinearity problems. Cook's distance (D) was used to detect outliers, and provides a criterion for deletion. A value greater than 1 is considered a probable outlier.

According to the study of Fitzpatrick et al. (27) ten possible combinations of horizontal and vertical alignment were considered. Table 5 contains the models that are finally proposed for Colombia. In total, they are 9 combinations, because the two cases of horizontal curve combined with a crest curve, with and without limiting sight distance, could be summarized in only one, since both equations provide similar results. In the cases of tangents combined with vertical sag curve or vertical crest curve without limiting sight distance was found, as in the study of Fitzpatrick et al. (27), that the operating speed corresponds with the desired speed in the section. The values of the coefficient of determination (R^2) are between 0.55 and 0.84. Table 5 includes the non Colombian models (Eq. 3, 4 and 5) that have been validated for Colombia.

Colombian models presented in Table 5 have been developed in horizontal curves with radii lower than 300 m. Models have been estimated with speeds collected in the midpoint of horizontal or vertical curves.

Fig. 1 shows the fit of proposed Colombian model for vehicle speeds on horizontal curve and grade between -9 % and -4 % (Table 5 and Eq. 6). The used sample was composed of 27 curves and the resulting coefficient of determination (R^2) is 0.72. In this model, independent variable is radius (r), not the inverse of radius (1/r), thus, a difference of other previous researches, the sign is positive and not negative. In consequence, very large horizontal curve radii could lead to extremely high values of predicted operating speed. This model could be used for radii lower than 300 m (maximum radius of curves in which speed data were collected).

$$V_{85} = 35.43 + 0.219\ r \qquad (6)$$

Similarly, the proposed Colombian model for operating speeds on horizontal curve and grade between 9 % and 4 % (Table 5 and Eq. 7) could be used for radii lower than 300 m.

$$V_{85} = 37.18 + 0.1\ r + 0.04\ r_{n-1} \qquad (7)$$

TABLE 5. Speed models proposed for Colombian highways

Horizontal and vertical alignments	Model	Sample	R^2	Origen
Horizontal curve; -9 % ≤ Grade < -4 %	$V_{85} = 35.43 + 0.219$ r	27	0.72	Colombia
Horizontal curve; -4 % ≤ Grade < 0 %	$V_{85} = 105.98 - 3709.90$ / r	25	0.76	Fitzpatrick et al.
Horizontal curve; 0 % ≤ Grade < 4 %	$V_{85} = 94.39 - 3188.66$ / r	29	0.79	Lamm et al.
Horizontal curve; 4 % ≤ Grade < 9 %	$V_{85} = 37.18 + 0.1\ r + 0.04\ r_{n-1}$	28	0.55	Colombia
Horizontal curve with sag vertical curve	$V_{85} = 102.70 - (730.39/r) - (1498.90/Lc)$	15	0.84	Colombia
Horizontal curve with crest vertical curve	$V_{85} = 93.79 - (867.61/r) - (935.62/Lc)$	32	0.66	Colombia
Sag vertical curve on horizontal tangent	V_{85} = Desired speed	-	-	Colombia
Vertical crest curve with non-limited sight distance on horizontal tangent	V_{85} = Desired speed	-	-	Colombia
Vertical crest curve with limited sight distance on horizontal tangent	$V_{85} = 105.08 – (149.69/K)$	9	0.60	Fitzpatrick et al.

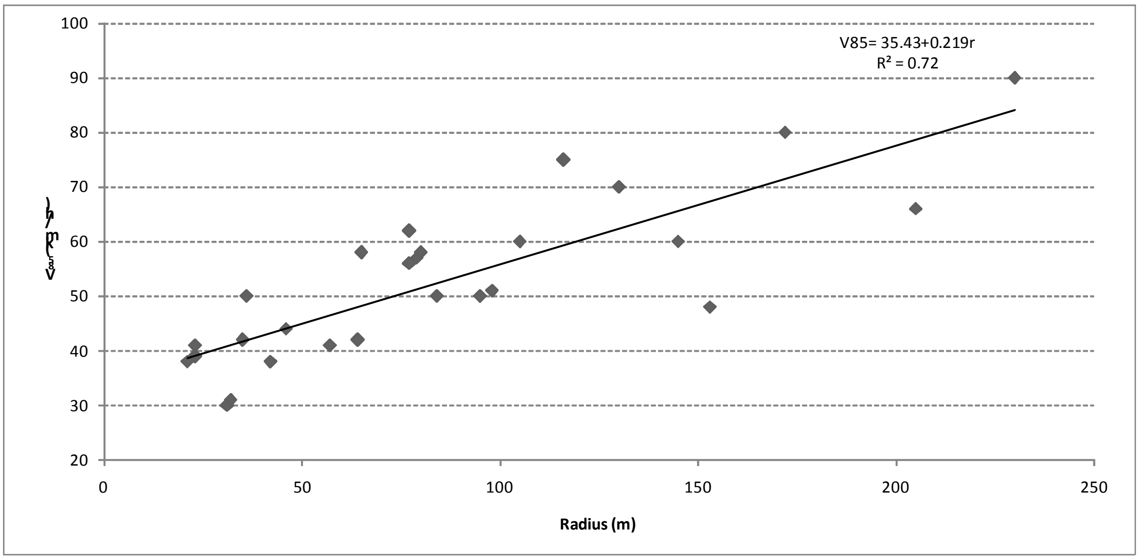
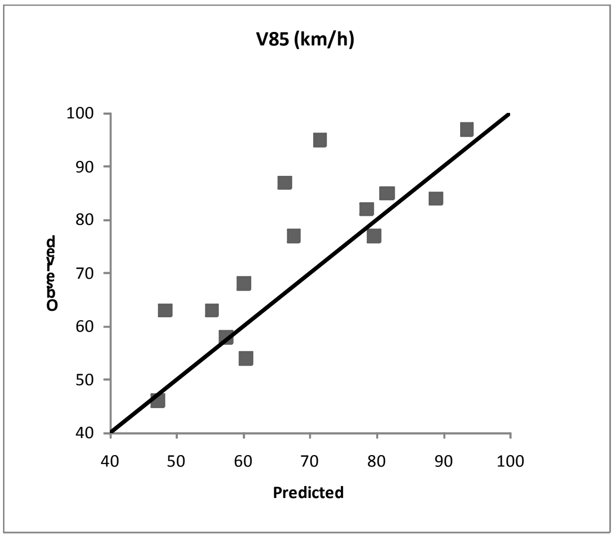


FIGURE 1 Colombian model. Prediction of operating speed for horizontal curve and grade between -9 % and -4 %

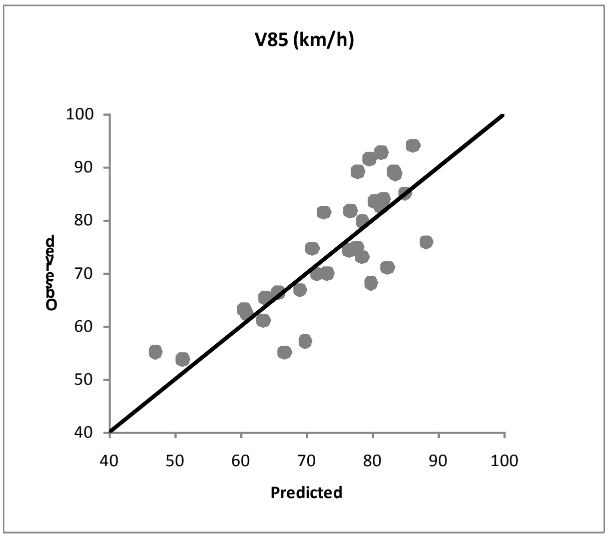
Figure 2 shows observed versus model predicted operating speed data for horizontal curves with sag (a) or crest (b) vertical curves (Table 5 and Eq. 8 and 9). In both cases, operating speed depends on inverse of radius and inverse of length of horizontal curve.

$$V_{85} = 102.70 - (730.39/r) - (1498.90/Lc) \qquad (8)$$

$$V_{85} = 93.79 - (867.61/r) - (935.62/Lc) \qquad (9)$$



(a)



(b)

FIGURE 2 Colombian model. Observed versus predicted operating speed for horizontal curves with sag (a) or crest (b) vertical curves

On the other hand, desired speed is reached when displacement is free of restrictions due to horizontal or vertical curvature, or limitations in the acceleration by proximity to restrictive elements. To determine desired speed data taken in tangents with length exceeding 200 m, and grades between – 4 % and 4 %, were used. In these circumstances, vehicles moving freely could achieve and maintain a constant speed.

Statistical analysis of the data measured on 14 sections that comply with the indicated features showed that, with a 95 % confidence level, the desired speed is between 92.85 km/h and 99.69 km/h. A value of 95 km/h, close to the mean, is recommended.

CONCLUSIONS

With a few exceptions, it is not recommended to apply in Colombia speed models developed by researchers in other countries, since the estimated values have significant differences with measured values. In some cases, speeds estimated by these models are negative, indicating that they are out of the range in which they were developed. Colombian speed models have been developed based on more than seventy sections of several highways considering different geometric combinations of horizontal and vertical alignment. Using these speed models and the applicable models from foreign researchers the consistency of most combinations of horizontal and vertical alignment could be analyzed.

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