Implementation of transition curves in vertical alignment of roads

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SUMMARY

Curves are used in the vertical alignment of roads to soften changes present on the grades. These curves are second order parabolas as they have been determined adequate. However, this can be improved. Therefore, the intention is to venture in the use of transition curves looking for improvements on the alignment that provide benefits to users, and provide the road with increased sight distance, smoothness and comfort.

The mathematical formulation that defines the transition, before and after the traditional parabolic curve, is presented in a concise manner identifying its geometric elements, and with applications to a real design case that shows the advantages, and also the possible inadequate uses, of these types of curves.

It is determined that transitioned vertical curves generally improve sight distance and, therefore, the safety of users, depending on site characteristics such as topography, grade changes, intertangency, and type of vertical curve. Nevertheless, care must be taken because complications may arise during the design and construction stages and this will eventually reduce sight distance and safety conditions due to inadequate designs by aesthetics, appearance and lack of continuity in the curvature.

This paper opens doors for new research. To be able to make new contributions to the development of the geometric design of roads, it is necessary to go deeper and make computer animated models that include the dynamics of vehicles when traveling on the road designed.

1. VERTICAL CURVES WITH SIMPLE PARABOLAS

Vertical curves are those that connect two consecutive tangents of the vertical alignment, their geometric form is that of a second order parabola: These curves are classified into two big groups: according to their grades – sag or concave and crest or convex, – and according to the curve’s longitude distribution – symmetric and asymmetric.-

The difference in the grades between the entrance tangent and the exit tangent is given by:

$$A = g_2 - g_1 \tag{1}$$

As it is a second degree curve, it has a constant grade change rate, r, given by:

$$r = \frac{A}{L_v} \quad (2)$$

Where L_v is the curve's longitude , and r is negative for crest curves and positive for sag ones. Figure 1 shows the form and the elements for a crest vertical.

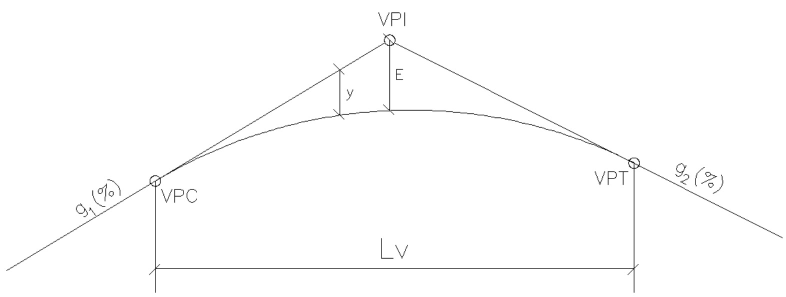

FIGURE 1 Elements of a parabolic vertical curve – Crest type.

The equation of the curve related to the X and Y axis located in the VPC is:

$$y = g_1 x + \frac{r}{2} x^2 \quad (3)$$

Where $(rx^2/2)$ is the vertical curve correction, noted as *y* in Figure 1.

2. TRANSITIONED VERTICAL CURVES

This document presents a methodology to design symmetric transitioned vertical curves using a third degree curve for the transition before and after the second degree parabolic curve. Deductions of the expressions are not presented because this topic is not the proposed objective.

Figure 2 illustrates a symmetric vertical curve where transition curves have been introduced, one after and one before the parabolic curve, and its geometric elements.

Each transition curve has a longitude l and a grade rate r_T which equals zero in the points of tangent - transition (TSV) y transition - tangent (STV) and equal to *r* at the points transition - curve (SCV) and curve – transition (CSV). A third degree polynomial offers that the transition curve gives a gradual change of r_T.

The formula which represents the transition curve with axis X and Y in the TSV, is:

$$y = ax + bx^2 + cx^3 \quad (4)$$

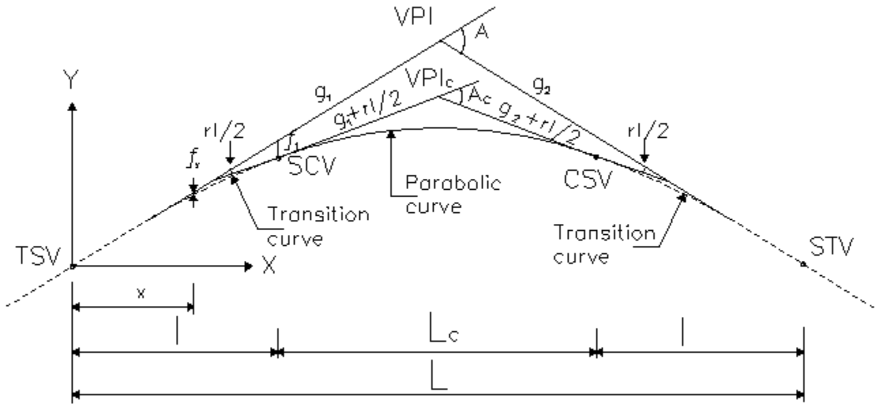

FIGURE 2 Transitioned crest vertical curve.

The vertical curve correction in the transition curve, the vertical distance f_x between the first tangent and the transition curve at any point x, is:

$$f_x = \frac{r}{6l}x^3 \quad (5)$$

$$r_T = {}^{rx}/_{l} \qquad 0 \leq x \leq l \quad (6)$$

The change in grade in the transition is increased gradually from zero in x = 0 to r in x = l. f_x , f_l and r_T are negative in crest curves and positive in sage ones.

The transition curve must have the same curvature, crest in this case, in order to guarantee continuity, and to also have the same grade in SCV and CSV for comfort.

Established as a premise is that the curve's SCV must be located $l/2$ further from the VPC that the initial curve would have had, meaning without transition, (Figure 3).

The angle of intersection between the first tangent and the tangent in SCV (or between the second tangent and the tangent in CSV), Δ_s is equal to $rl/2$ (see Figure 3), which is:

$$\Delta_s = |p_2 - p_1| \qquad \Delta_s = rl/2 \quad (7)$$

It is possible to demonstrate, for a symmetric curve, that the vertical point of intersection, VPIc, between the tangents in SCV and CSV is in the Middle of the curve, and in the same abscissa of the VPI.

The grade change for the displaced parabolic curve (A_c) is the difference between the grade in CSV: ($g_2 - rl/2$) and the grade of the tangent in SCV: ($g_1 + rl/2$), as follows:

$$A_c = A - r * l \quad (8)$$

When $r = A_c/L_c$, the longitude of the parabolic curve L_c is the result of:

$$L_c = \frac{A}{r} - l \tag{9}$$

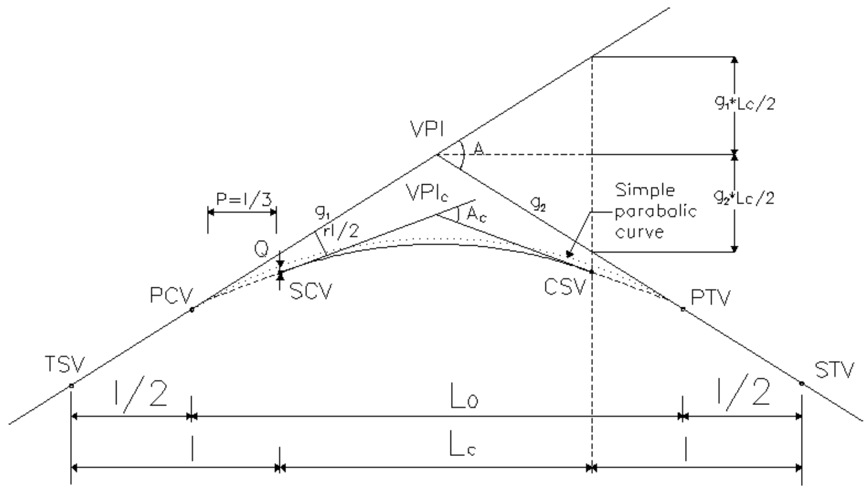

FIGURE 3 Geometric characteristics of a transitioned crest vertical curve.

The total longitude of the vertical curve of L transition, for a symmetric curve, is:

$$L = L_c + 2l \tag{10}$$

The transition curve is defined when A, r y l are given. Defined as r is the inverse of K, a value that can be obtained in design norms, according to sight distance, design velocity and A; K is the longitude of the curve for each grade change of 1 %, as follows: r = 0,01/K.

2.1 Longitude of the Transition Curve

When a vehicle takes a vertical curve, centrifugal forces in VPC develop and cause nuisances, especially in sag curves, where the centrifuge force and gravity are in the same direction.

The change in the centripetal acceleration, with regards to time (C) is da_T/dt, or $da_T/dx * dx/dt$, enabling to demonstrate that:

$$C = rv^3/l \tag{11}$$

Therefore, the minimum longitude of the transition curve is: $l = \frac{rv^3}{C}$

If r is expressed in terms of K (r = 0.01/K), and velocity in km/h terms, the equation is:

$$l = 2.15 * 10^{-4} * v^3/KC \tag{12}$$

For horizontal curves the maximum value for C is from 1 to 3 feet/s^3 (0,3 to 1,0 m/s^3) (AASHTO, 2004). However, the maximum centripetal acceleration permitted for vertical curves is 0.3 m/s^3, therefore the value of C corresponding to horizontal curves is high. For this reason trials to estimate the maximum value of C in vertical curves are required. In Tables 1 and 2 present the minimum longitude of transition for crest and sag curves corresponding to C = 0,05 and 0,10 m/s^3 and the range of K values provided by design guides and based on the requirements for stopping sight distance (AASHTO, 2004).

TABLE 1 Minimum Longitude of Transition for Crest Vertical Curves

Design speed (km/h)	K (m)		$l_{minimum}$ (m)			
			$C = 0.10\ m/s^3$		$C = 0.05\ m/s^3$	
	Low	High	Low	High	Low	High
30	3	3	20	20	39	39
40	5	5	28	28	55	55
50	9	10	30	27	60	54
60	14	18	34	26	67	52
70	22	31	34	24	67	48
80	32	49	35	23	69	45
90	43	71	37	23	73	45
100	62	105	35	21	70	41
110	80	151	36	19	72	38
120	102	202	37	19	73	37

TABLE 2 Minimum Longitude of Transition for Sag Vertical Curves

Design speed (km/h)	K (m)		$l_{minimum}$ (m)			
			$C = 0.10\ m/s^3$		$C = 0.05\ m/s^3$	
	Low	High	Low	High	Low	High
30	4	4	15	15	29	29
40	8	8	18	18	35	35
50	11	12	25	23	49	45
60	15	18	31	26	62	52
70	20	25	37	30	74	59
80	25	32	44	35	89	69
90	30	40	53	40	105	79
100	37	51	59	43	117	85
110	43	62	67	47	134	93
120	50	73	75	51	149	102

With regards to drainage: there is a small road section with null grade (horizontal) in vertical curves with grade changes from positive to negative. In such case it is possible to have good drainage if a grade of at least 0,25% extends for more than 15 meters beyond the point of grade change - the highest point in crest curves and the lowest point in sag curves - (AASHTO, 2004). In a simple vertical curve this condition is met if $K \leq 51m$ $(r \geq 0.000196)$. A transitioned vertical curve, a total longitude L that meets this condition is obtained by applying the minimum value of r in equation (9) and observing that L $= L_c + 2l$, however, this longitude is valid if the point of grade change occurs in a parabolic curve. If the change of grade occurs in the transition curve, the maximum longitude of the vertical curve for drainage to be appropriate is calculated using the same guide, with a minimum slope of 0,25 % at 15 meters from the level point. When the curvature changes gradually, the strongest case occurs when the required grade change is considered in the flattest part of the curve, which is adjacent to the tangent. The following must be followed:

$$r \geq 2{,}222 * 10^{-5}\,l \quad (13)$$

In the case of a point of grade change that occurs in the transition curve, drainage is a problem only at the flattest part of the curve, if the longitude of the curve is more than the maximum longitude for the drainage, whose extension depends on the curvature and longitude of the curve. However, this drainage guide is not proposed as maximum design, but as the value past which drainage must be designed with more care (AASHTO, 2004). Therefore, if the curve's longitude by sight distance is larger than the longitude by drainage, a higher value should be used to assure the appropriate drainage of the surface.

The location of the highest, or the lowest, point (crest or sag curve) is important due to drainage inconveniences when grade is zero for a road section, or when a sag curve has no drains. The station of the highest (lowest) point, x_0, is:

$$x_0 = \frac{l}{2} - \frac{g_1}{r} \qquad l \leq x_0 \leq l + L_c \quad (14)$$

$$x_0 = \left(-2 * g_1 * \frac{l}{r}\right)^{1/2} \qquad x_0 \leq l \quad (15)$$

$$x_0 = L - \left(\frac{2 * g_2 l}{r}\right)^{1/2} \qquad x_0 \geq l + L_c \quad (16)$$

2.2 Layout of the Curve

In order to draw the vertical transition curve, it becomes necessary to determine the elevations of the curve in different points. The elevation and the station of the VPI are defined by the drawing of the tangents; and the parameters A, r and l or (A, l and L_c) are determined as previously described. The stations and elevations of the principal points that define the transitioned vertical curve are given by:

Station TSV = Station VPI – $(l + L_c/2)$ (17)
Station STV = Station TSV + $(L_c + 2l)$ (18)
Station SCV = Station TSV + l (19)
Station CSV = Station TSV + $l + L_c$ (20)
Elevation TSV = Elevation VPI - $g_1(l + L_c/2)$ (21)
Elevation STV = Elevation VPI + $g_2(l + L_c/2)$ (22)
Elevation SCV = Elevation TSV + $g_1 l + rl^2/6$ (23)
Elevation CSV = Elevation STV - $g_2 l + rl^2/6$ (24)

A summary of the principal formulas to be used in the calculation of the vertical alignment on curves using transitioned curves is presented next. Likewise, Figures 2 and 3 show the principal elements in the calculation of these curves.

The elevations in the first transition curve and the parabolic curve are equal to: Elevation TSV +y_x.

Calculation between TSV and SCV:

$$y = g_1 x + rx^3/6l \qquad 0 \leq x \leq l \quad (23)$$

Where the vertical curve correction (VCC) is $f_x = rx^3/6l$ defined in equation (5).

Calculation between SCV and VPI:

$$y = \left(g_1 l + \frac{rl^2}{6}\right) + \left(g_1 + \frac{rl}{2}\right) * (x - l) + \frac{r}{2}(x - l)^2 \qquad l \leq x \leq l + L_c \tag{24}$$

The vertical curve correction at point x (with x measured from TSV) of the vertical curve of the first tangent f_x is given by:

$$CCV = f_x = y - g_1 x \qquad l \leq x \leq l + L_c \tag{25}$$

Calculation between VPI and CSV:

$$CCV = -g_1(l + L_c/2) + y + g_2 x \tag{26}$$

With y of equation (24) and x measured from the VPI.

Calculation between CSV and STV:

Elevations in the second transition curve must be calculated as: Elevation ETV + y_x.

In equation (23), x is measured from STV , and g_1 is replaced by g_2 as follows:

$$y = -g_2 x + rx^3/6l \tag{27}$$

Where the vertical curve correction is: $VCC = f_x = rx^3/6l$ with $l + L_c \leq x \leq 2l + L_c$ as defined by equation (5).

Finally, the elevation for any station is: $Elevation = Tangent\ elevation + CCV$

3. APPLICATION OF TRANSITIONED VERTICAL CURVES TO A REAL CASE

An application exercise of this methodology, to identify its applicability and advantages, was conducted on 5 km in the road Angelópolis – Caldas (Department of Antioquia)'s geometric design, which have a total longitude of 15 km and a design speed of 30 km/h.

The methodology applied is conventional with regards to terrain surface, horizontal alignment, superelevation and even in the obtaining of the longitudinal profile. Differences appear in the design of the vertical alignment, the calculation of the elevations, and therefore in the transversal sections and measurement of earthworks.

Crest vertical curve: It corresponds to VPI 7 located in station km 0+502,19, with elevation 1938,212, the vertical curve is designed in a traditional manner and transition curves are applied. Values for K and l are selected from Table 1; the longitude of the vertical transition curve is selected in a conservative manner based on the value of K. The principal results of the complete transitioned curve are shown in Table 3.

TABLE 3 Elements of a Transitioned Crest Vertical Curve

Station VPI 7	km 0+502,19	Elevation VPI	1938,212
Design speed (km/h)	30	Type	Crest vertical
g1	0,09742	r = 0.01/K	0,00333
g2	-0,09364	r	-0,00333
K (m) (Table 1)	3		
l (m) (Table 1)	20	Lc (m) (Eqn. 9)	37,32
Elevation TSV (m) (Eqn. 21)	1934,45	Station TSV (Eqn. 17)	k 0+463,53
Elevation STV (m) (Eqn. 22)	1934,59	Station STV (Eqn. 18)	k 0+540,84
Elevation SCV (m) (Eqn. 23)	1936,17	Station SCV (Eqn. 19)	k 0+483,53
Elevation CSV (m) (Eqn. 24)	1936,24	Station CSV (Eqn. 20)	k 0+520,84

In Table 4 is showed the results to vertical alignment calculation by the axis of a crest vertical curve case (VPI 7), on its characteristic curve's stations and other stations multiples of 10 m.

TABLE 4 Calculation of the Vertical Alignment of a Transitioned Crest Vertical Curve

Point	Station	y_X	Elevation on Tangent	VCC	Elevation on curve
TSV	k0+463,53	0,00	1934,45	0,00	1934,45
	k0+470,00	0,62 (Eqn. 23)	1935,08	-0,01 (Eqn. 5)	1935,07
	k0+477,19	1,26 (Eqn. 23)	1935,78	-0,07 (Eqn. 5)	1935,71
	k0+480,00	1,48 (Eqn. 23)	1936,05	-0,12 (Eqn. 5)	1935,93
SCV	k0+483,53	1,73 (Eqn. 23)	1936,39	-0,22 (Eqn. 5)	1936,17
	k0+490,00	2,07 (Eqn. 24)	1937,02	-0,51 (Eqn. 25)	1936,52
	k0+500,00	2,33 (Eqn. 24)	1938,00	-1,22 (Eqn. 25)	1936,78
VPI Nº 7	k0+502,19	2,34 (Eqn. 24)	1938,21	-1,42 (Eqn. 25)	1936,79
	k0+510,00	2,25 (Eqn. 24)	1937,48	-0,78 (Eqn. 26)	1936,70
	k0+520,00	1,85 (Eqn. 24)	1936,54	-0,25 (Eqn. 26)	1936,29
CSV	k0+520,84	1,80 (Eqn. 24)	1936,46	-0,22 (Eqn. 5)	1936,24
	k0+527,19	1,21 (Eqn. 27)	1935,87	-0,07 (Ecn. 5)	1935,80
	k0+530,00	0,98 (Eqn. 27)	1935,61	-0,04 (Ecn. 5)	1935,57
	k0+540,00	0,08 (Eqn. 27)	1934,67	0,00 (Ecn. 5)	1934,67
STV	k0+540,84	0,00 (Eqn. 27)	1934,59	0,00 (Ecn. 5)	1934,59

Figure 4 shows the difference in the VPI 7 between the conventional curve (parabolic only) and the transitioned vertical curve.

For the crest vertical curve of VPI 7, the stopping sight distance was improved by 6,5 % with regards to the distance obtained in a traditional curve, 33,7 m. This additional distance provides more safety when braking.

The same procedure and formulas are used to calculate elements and the vertical alignment for a sag vertical curve, the only difference is that r is positive in this case.

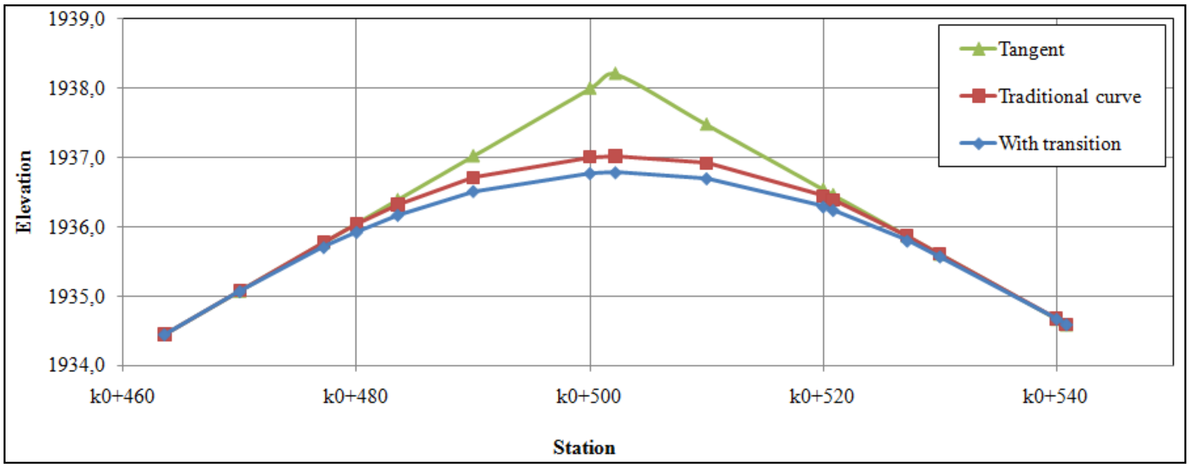

FIGURE 4 Crest vertical curve at VPI 7.

Sag vertical curve: Corresponds to VPI 8 in km 0+572,51 and elevation 1931,60. K and l values are selected from Table 2. The longitude of the vertical transition curve is selected in a conservative manner based on the value of K. The results obtained are shown in Table 5.

TABLE 5 Elements of a Transitioned Sag Vertical Curve

Station VIP 8	km 0+572,51	Elevation VPI	1931,60
Design speed (km/h)	30	Type	Sag vertical
g1	-0,09364	r = 0.01/K	0,0025
g2	-0,02339	r	0,0025
K (m) (Table 2)	4		
l (m) (Table 2)	15	Lc (m) (Eqn. 9)	13,1
Elevation TSV (m) (Eqn. 21)	1933,64	Station TSV (Eqn. 17)	k0+550,96
Elevation STV (m) (Eqn. 22)	1931,12	Station STV (Eqn. 18)	k0+594,06
Elevation SCV (m) (Eqn. 23)	1932,33	Station SCV (Eqn. 19)	k0+565,96
Elevation CSV (m) (Eqn. 24)	1931,57	Station CSV (Eqn. 20)	k0+579,06

Table 6 presents the elevations by the axis of this transitioned sag vertical curve (VPI 8), in the stations that define the alignment and other stations multiples of 10 m.

TABLE 6 Calculation of the Vertical Alignment of a Transitioned Sag Vertical Curve

Point	Station	y_X	Elevation on Tangent	VCC	Elevation on curve
TSV	k0+550,96	0,00	1933,64	0,00 (Eqn. 5)	1933,64
	k0+557,51	-0,61 (Eqn. 23)	1933,03	0,01 (Eqn. 5)	1933,04
	k0+560,00	-0,83 (Eqn. 23)	1932,80	0,02 (Eqn. 5)	1932,82
SCV	k0+565,96	-1,31 (Eqn. 23)	1932,24	0,09 (Eqn. 5)	1932,33
	k0+570,00	-1,59 (Eqn. 24)	1931,86	0,19 (Eqn. 25)	1932,05
VPI Nº 8	k0+572,51	-1,75 (Eqn. 24)	1931,63	0,27 (Eqn. 25)	1931,90
CSV	k0+579,06	-2,08 (Eqn. 24)	1931,47	0,09 (Eqn. 26)	1931,57
	k0+580,00	0,41 (Eqn. 27)	1931,45	0,08 (Eqn. 5)	1931,53
	k0+590,00	0,10 (Eqn. 27)	1931,22	0,00 (Eqn. 5)	1931,22
STV	k0+594,06	0,00 (Eqn. 27)	1931,12	0,00 (Eqn. 5)	1931,12

Figure 5 shows the difference in the VPI 8 between the conventional curve and the transitioned vertical curve.

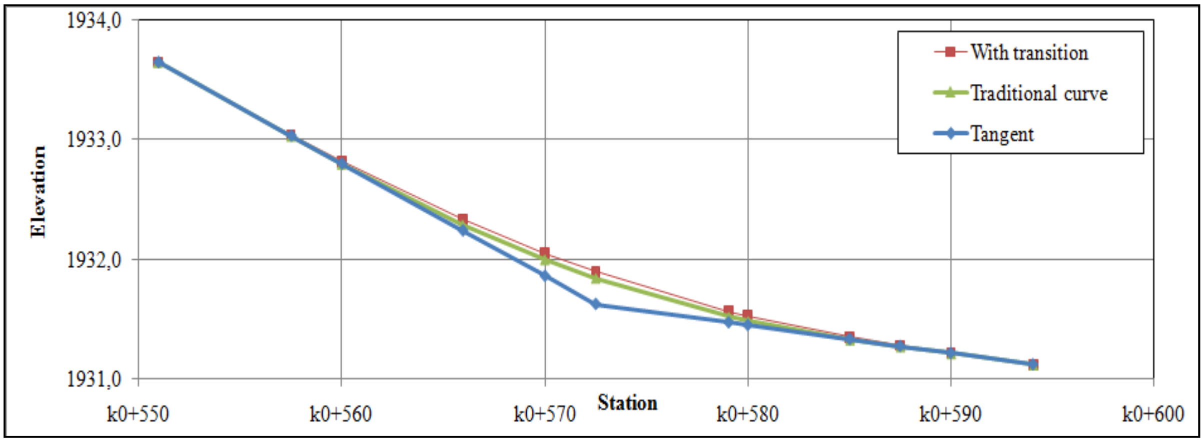

FIGURE 5 Sag vertical curve at VPI 8.

Additional examples: The desired benefits of applying the transition aren't obtained in some curves, where the traditional alignment is more favorable, for instance:

VPI 23 case (crest vertical curve): when applying the minimum transition curve ($l = 20m$) the total longitude of the vertical curve is $L = 45{,}42\ m$, shorter than the one used in traditional design of $L_{cv} = 60m$, in this manner the transitioned curve is located above and the desired benefits are not obtained. In this case, the long transition ($l = 39\ m$), cannot be applied because $L_c < 0$ and this presents a curve discontinuity which disagrees with the criteria for comfort. (see Figure 6).

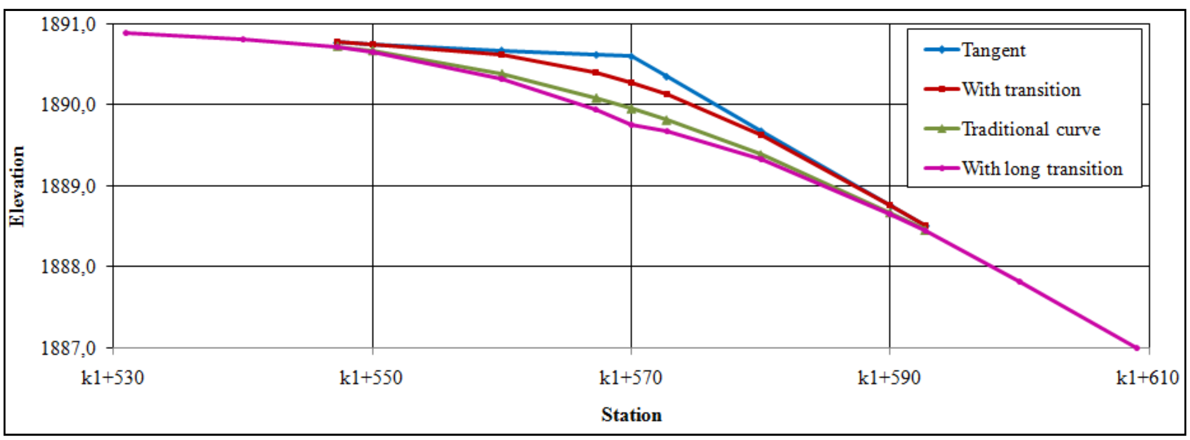

FIGURE 6 – Crest vertical curve at VPI 23.

VPI 30 case (sag vertical curve): see Figure 7, the transitioned curve ends up lower than the traditional one which is not favorable for these types of curves. This is because the traditional curve has a longitude much larger with regards to the minimum of 70 m, while the longest transitioned curve with $l = 29m$ reaches a longitude of $L = 82{,}66m$, which, being higher than the traditional, is not enough to provide the expected benefits.

In other cases it is impossible to apply the long transition curve due to space restrictions as it would overlap with the next, or with the previous, curve.

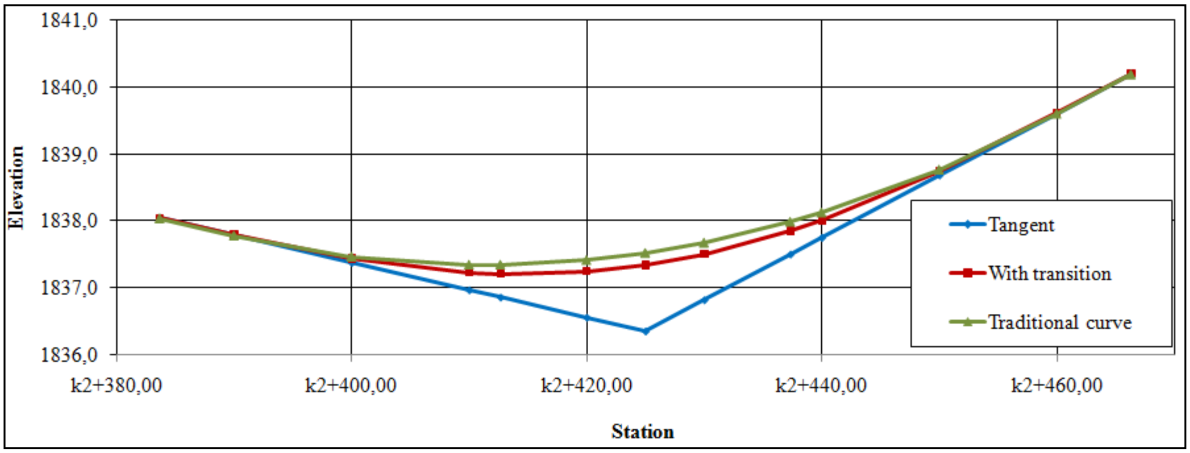

FIGURE 7 – Sag vertical curve at VPI 30.

4. IMPLICATIONS OF THE USE OF TRANSITIONED VERTICAL CURVES IN THE ROAD'S GEOMETRIC DESIGN

4.1 Sight Distance

Some crest vertical curves restrict the driver´s sight distance which enables it to observe a situation that would require stopping. Concerning distance for overtaking or passing other vehicles, these require very large longitudes of sag vertical curve. Night sight distance is considered for sag vertical curves where vehicle´s headlights are important. Daylight sight distance is not a problem unless there is coincidence horizontal and vertical curves or an obstacle is located above the road such as bridge that restricts sight distance.

Transitioned vertical curves, in general and if they are possible to implement, improve sight distance conditions and therefore provide better safety conditions. However many other factors are involved in safety.

4.2 Earthworks and Constructions Limits

Generally, earthworks are increased in crest vertical curves if it concerns cuts, but it decreases if it is embankments. The opposite happens in sag vertical curves; therefore a single behavior concerning an increase or a decrease of earthworks cannot be established. The land requirement for the project is not affected by the implementation of transitions in vertical curves, as the construction limits do not considerably change.

5. CONCLUSIONS AND RECOMMENDATIONS

This paper presents the methodology and geometric calculations of a symmetric vertical curve upon which a transition third degree curve is introduced which enables the gradual change of the curvature rate. The distance from the start of the transition curve (TSV) and the start of the original curve (VPC), as a premise, is established to be half of the transition curve.

Minimum curve longitude requirements are established based on comfort by assuming typical values of the maximum rate of change of centripetal acceleration. Research must be conducted for the latter to determine its real values.

A range for the transitioned vertical curve must be established, and then elect a value that is in accordance with the site´s space characteristics, grade changes and intertangencies. One may say that L_{min} is the value determined during the development of this paper (presented in Tables 1 and 2), while L_{max} is obtained when $L_c = 0$.

If traditional design considers the minimum longitude of vertical curves, the use of transition curve is favorable as it improves sight distance and safety. However, if the design with simple parabola uses a longitude that is longer than minimum, transitioned curves are not always an adequate choice. When the transitioned curve is L_{max} and is less than the minimum required by specifications, design must be done with traditional parabola.

A vertical alignment design applying transitioned vertical curves softens the effect of the centrifuge force generated when going from a tangent to a parabola in traditional design, causing more nuisances in sag vertical curves where gravity and centrifuge force work in the same direction. A smoother change is enabled through the transition which results in more comfort.

It is not determined whether or not there are savings considering earthworks as this depends on the projects´ specific characteristics. For this case, a specific study which considers not only costs increases, but whether or not transitions are worth the effort for users´ comfort and safety, is recommended.

The staking and construction with these type of curves are not considered affected if compared with the traditional method as the unevenness between the terrain (terrain level or black elevation – term used in Colombia and other countries) and the leveled area defined by the calculation of the vertical alignment (road level or red elevation - term used in Colombia and other countries) are placed on the stakes.

6. REFERENCES

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