Selection of 3D Elements for Different Speeds in the 3D Modeling of Highways

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April 2010

4,204 words+5 Figures+5 Tables=6,704 words equivalent

Revised paper submitted for presentation at the 4th International Symposium on Highway Geometric Design
Valencia, Spain
June 2-5, 2010

Abstract

Traditionally, geometric design of roadways is a two-stage process in which the horizontal alignment is designed first and then the vertical alignment is designed based on the former. Separate processing of alignments prevents any corrections to be made in the road design for issues related to sight distance and other geometric inconsistencies since the three-dimensional (3D) picture of the road, which enables road users to adapt to their driving style in accordance with highway geometry, is not obtained until the full 3D view of the roadway based on the superimposition of horizontal and vertical alignments is obtained. Due to the mathematical complexities associated with the 3D representation of highways, limited literature is available on 3D geometric design methodology of highways. In our previous works a new mathematical method was introduced in which the traditional two-stage design process was reduced to a single-stage design process by representing a 3D road surface with 3D elements, such as 3D tangent lines and piecewise polynomials (splines). The single-stage design process involved identification of the control points on a given terrain, through which the roadway would pass, and finding suitable 3D tangent lines and/or splines to connect those control points. The splines were represented by a 4^{th} degree piecewise polynomial equation. In this paper we investigate how the characteristics of the 3D elements change if one was interested in designing a 3D road for different speeds. Due to the mathematical limitations of the polynomial functions in investigating effects of varying speeds on the 3D alignment we introduce a parametric equation to represent the 3D elements. Using the proposed parametric equation we show the variation in speed and radius with the coefficient of the parametric equation.

Key-words: 3D Geometric Design of Highways, splines, horizontal alignment, vertical alignment

Introduction

Traditional geometric design of roadways involves the design of horizontal and vertical alignments and their superposition for a smooth and safe driving condition. Some critical information for geometric design of roadways may not be available in horizontal alignment or in vertical alignment because of the planar projection of the roadway onto XY plane or XZ plane (Cartesian coordinate system). For example, a designer may not be able to adjudge the sight distance by looking at horizontal or vertical alignment separately, because in horizontal alignment the elevation information is not available and in vertical alignment the horizontal alignment of road is not available. Another problem that can be seen in 2D geometric design method is that the designer may not be able to identify the illusion effect when a horizontal curve appears simultaneously with a vertical curve. Without identifying such illusions and other geometric inconsistencies due to the separate processing of horizontal and vertical alignments it is possible that there may be driver discomfort or limited sight distance availability after the roadway is constructed. These problems, to a large extent can be solved by designing highways using three dimensional techniques (*1, 2*). Using these techniques the designer can view the alignment three dimensionally and identify the errors that cannot be identified through the separate processing of horizontal and vertical alignments at the onset. It is also possible to exploit 3D visualization techniques to address sight distance and other geometric design inconsistencies (*3-5*) in geometric design of highways.

In this paper we briefly describe our previous method for representing a 3D highway alignment using 3D elements, such as 3D tangent lines and piecewise polynomials (splines). While the 3D tangent lines and splines are able to represent a 3D alignment, due to the mathematical limitations of the polynomial function it is difficult to investigate the effects of varying speeds on the 3D elements which make up the alignment. Therefore, we exploit a substitute parametric equation to represent the 3D alignments, which offers a mathematically convenient option to investigate the effects of varying speed on 3D elements.

Literature Review

The geometric elements of horizontal and vertical highway alignments have been discussed in various articles (Mannering et al. 2005; Wright and Dixon 2005) and dates back to early 1900. A horizontal alignment is composed of tangent and circular curve sections while vertical alignments are composed of tangent and parabolic sections. Transition curves, such as Euler Clothoid curves are introduced between tangent and circular sections of horizontal alignments, to allow gradual and smooth transition to and from circular curves. Due to the complex mathematical equations of such curves, their precise treatment is often ignored when developing highway design plans. Instead, highway agencies generally use trial and error for fitting arbitrary curves to transition to/from circular sections. Limited efforts have been made towards the 3D treatment of highway alignments, to date. Many articles are not available in English. For example, we found some articles (*8-13*) dealing with the 3D geometric design methodology in

German, Chinese, and Japanese. One of the key papers published in English language dealing with a true 3D-design methodology was authored by Kuhn (*2*) that was presented at the 3rd International Symposium on Highway Geometric Design. In that paper an extensive review of the literature dealing with previous 3D design methodology was provided. In this paper, some background literature that provides a good basis for 3D geometric design methodology is taken from Kuhn (2), which is briefly described next.

Brauer (*8*) turned his attention to the differential geometric principles of mathematical functions in relation to 3D route planning for roads for the first time in 1942. A “moving trihedron,” consisting of the normal vector of the tangent, the binormal vector situated in a vertical relationship to the former and lying on the parallel wheel axis and the unit normal vectors situated in a vertical relationship to both, was defined. Lorenz (*9*), in his investigations into 3D route planning, determined the axis with the help of cylindrical barrels. These cylindrical barrels served as supports for the 3D transition bend, which was designed to be interposed between two skew lines g1 and g2, which ran tangentially to the barrel. Using his Freising model, Freising (*10*) proposed a geometrical design system, where the route planning axis consisted of even 3D curves. For this purpose the weak bend was used as the 3D element. Scheck (*11*) examined various approaches for optimizing 3D routes. This involved a gradual optimization of the route plan in the horizontal and vertical alignment plans. Based on a preferred option in the horizontal alignment plan, the vertical alignment plan was calculated and determined using dynamic optimization. However, this iterative horizontal and vertical alignment plan optimization did not depict any 3D route-planning system. Borgmann (*12*) examined an interpolation of 3D fixed points using a flexible ruler for a specific case, where the hyperbolic transition bend was used as a 3D bend. Psarianos (*13*) carried out extensive research into developing a model representation using the 3D design elements of a straight, a helix and a choroclothoid. In that application the chloroclothoid was used as a transition bend between the straight and the helix.

Hassan et al (*14*) investigated the effects of 3D alignment on the design requirements for sight distance. Makanae (*15*) developed a 3D alignment design system in the virtual space recreated by stereoscopy of aerial photographs. In order to define the 3D alignment in the virtual space, B-spline curves were applied. In another publication, Makanae (*16*) applied parametric curves for representing highway alignments. Several other applications (*8-13*) in different languages (including Japanese, Chinese, and German) were found dealing with 3D representation of highway geometry.

The Federal Highway Administration (FHWA) (*17-19*) developed an Interactive Highway Safety Design Model (IHSDM) to check for design consistency using empirical data. In recent years, researchers have recognized that proper coordination of different elements of horizontal and vertical alignments was especially desired for driver comfort, drainage, and safety (*20-23*). Bulk of the recent research works for horizontal and vertical alignment coordination (*20-26*) have been focused in safety and sight-distance aspects. Jha et al. (*27*) developed 3D highway alignment optimization models while ensuring design consistency.

While exhaustive treatment of 3D highway alignment is found in the literature as noted above, issues related to a single stage road design procedure still remain that need further investigation. For example, none of the existing methods provide a unified framework for superimposing horizontal and vertical alignments as well as cross sections in a manner that minimizes design inconsistency and associated errors.

While Kuhn (*2*) presented a 3D geometric design method for roadways using a combination of fixed, dialogue and coupling elements, he did not provide enough information on the choice of elements. Our recently published paper in the Urban Transport conference (*1*) seems to be the latest available article on 3D geometric design methodology in which 3D tangent lines and piecewise polynomials (splines) are used to represent a 3D roadway. This method offers an improvement over Kuhn's (2) method since Kuhn's method is computationally inefficient in representing 3D alignments when the number of fixed, dialogue, and coupling elements increase. Moreover, it does not allow investigation of varying speeds on the 3D elements.

The Proposed 3-D Design Methodology

The methodology for 3-dimensional roadway design is based on the existing 2-dimensional horizontal and vertical design methods. In 2-D design methods, the roadway alignment is composed of design elements which are a combination of straight lines, circular curves, parabolic curves, and spiral curves. In either horizontal or vertical alignment, a change in straight road (straight line) direction is incorporated through a curve. i.e., two straight portions of a road which are adjacent and have different directions are connected with a curve. Circular curves are used in horizontal alignment design and parabolic curves are used in vertical alignment design, to join the straight portions. A similar approach is used for three dimensional roadway designs.

Design Elements

In our approach the alignment is composed of two design elements, 3-D straight lines and 3-D spline curves. The 3D straight lines are connected with spiral curves.

3-D Straight Lines

A 3-D straight line consists of a start and an end point. In Cartesian coordinate system the start point is represented as (x_s ,y_s, z_s) and the end point is represented as (x_e, y_e, z_e). The location of the start and end points gives the direction of the straight line. A typical 3-D straight line is shown in Figure 1.

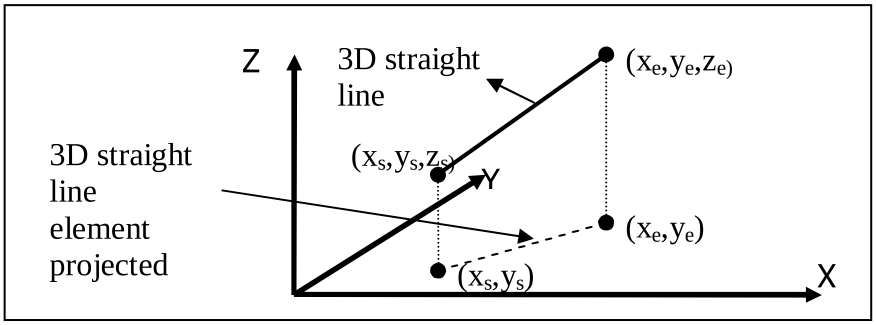

Figure 1: A typical 3D straight line.

3-D Spline curves

Given a set of control points through which the designer wants the alignment to pass, polynomial curves can be used to pass through the control points. But, there are two disadvantages of using polynomial curves:

1. There will be no control on the adjustment of the curve between two control points for design purposes.
2. As the number of control points increases, the order of the polynomial increases and hence number of coefficients of the polynomial also increases.

Instead of using one polynomial to pass through control points, one can use piecewise polynomials to reduce above said problems. These piecewise polynomials together are termed as spline curves. Splines form a good fit for given control points and intermediate points can be interpolated easily (*26, 27*). In the proposed design approach, cubic splines (of order 4) are used.

Let C = {0, 1, 2, .. l-1} denote l knots (which are control points in this methodology and a part of the total set of control points). These knots are in a sequence. Let $c \in C$. c = 0 denotes the first knot in the sequence and $c = l$-1 denotes the l^{th} knot. Let (x_c, y_c, z_c) be the Cartesian coordinates of a knot in C. Let $X = \{x_0, x_1, .. x_{l-1}\}$ be the set of x coordinates of all the l knots. Let $Y = \{y_0, y_1,.. y_{l-1}\}$ be the set of y coordinates of all the l knots. Let $Z = \{z_0, z_1, .. z_{l-1}\}$ be the set of z coordinates of all the l knots. As explained earlier, we connect these l knots with a spline. The spline will have l-1 piecewise polynomials.

The parametric functions of p^{th} piecewise polynomial of a spline that is between knots c and $c+1$ is given below.

$$x_t^p = a_{px}t^3 + b_{px}t^2 + c_{px}t + d_{px} \quad (1)$$

$$y_t^p = a_{py}t^3 + b_{py}t^2 + c_{py}t + d_{py} \quad (2)$$

$$z_t^p = a_{pz}t^3 + b_{pz}t^2 + c_{pz}t + d_{pz} \quad (3)$$

where

$$t \in [c, c+1) \qquad (4)$$

$p = c+1$. p denotes the p^{th} polynomial piece of the spline that is between knots c and $c+1$. For example $p = 2$ denotes the second polynomial of the spline that is between the knots 1 and 2.

x_t^p = p^{th} piece x coordinate considered at position t.
y_t^p = p^{th} piece y coordinate considered at position t.
z_t^p = p^{th} piece z coordinate considered at position t.

$a_{px}, b_{px}, c_{px}, d_{px}$ are coefficients of the p^{th} polynomial piece in X
$a_{py}, b_{py}, c_{py}, d_{py}$ are coefficients of the p^{th} polynomial piece in Y
$a_{pz}, b_{pz}, c_{pz}, d_{pz}$ are coefficients of the p^{th} polynomial piece in Z

In the literature, many algorithms are available which can be used to get the coefficients of piecewise polynomials. Examples include cardinal splines, Catmull Rom splines, and Kochanek Bartels Splines (*28-30*). A typical 3D spline is shown in Figure 2. For more details on splines, the readers may refer to any computer graphics book on using splines for graphics.

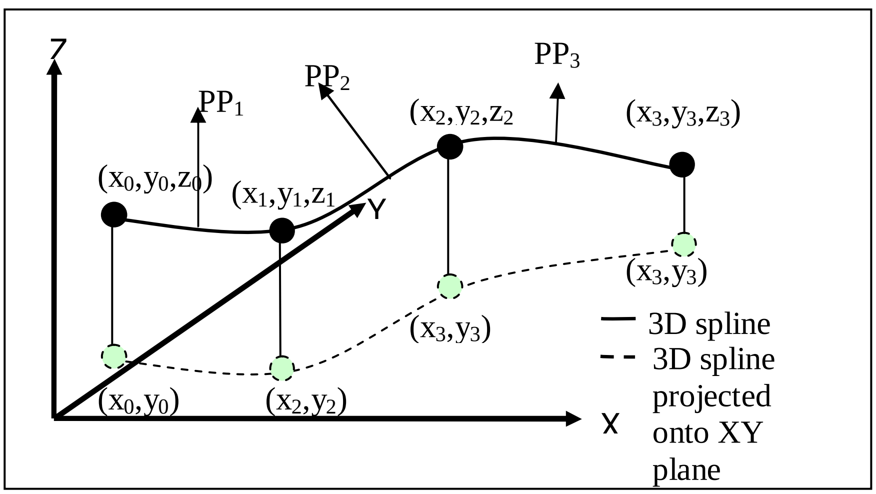

Figure 2: A typical 3D spline. Here 3D spline has three piecewise polynomials PP_1, PP_2, PP_3 with four knots.

Design Procedure

Given a 3-D terrain, the designer has to select some control points over the terrain through which the alignment should pass. Keeping these control points in view, the designer best fits 3D straight lines based on the control points' positions. The number of 3D straight line design elements depends on the site conditions. Then the designer fits a spline element that passes through control points that are between two consecutive straight line elements. The designer can change the shape of the spline by adding additional control points or by changing the position of the existing control points. Then the designer needs to check the alignment for technical and safety issues.

An Example

An alignment created using existing methods

A 3D roadway alignment is created using existing methods for experimental purpose. The 3D alignment created has three design elements namely straight-lines, clothoids, circular curves in the horizontal alignment and straight-lines, and parabolic curves in the vertical alignment. The geometric details of the alignments are given below.

Horizontal alignment geometric details

Table 1: Geometric details for horizontal alignment

Type	Start		End		Length (m)	Radius(m)
	x	y	X	y		
Linear	17612.47	22314.02	17873.51	22102.24	336.14	
Clothoid	17873.51	22102.24	17905.12	22077.74	40	
Circular	17905.12	22077.74	18118.9	22025.55	225.32	-300
Clothoid	18118.9	22025.55	18158.24	22032.73	40	
Linear	18158.24	22032.73	18355.63	22073.32	201.51	
Clothoid	18355.63	22073.32	18394.97	22080.5	40	
Circular	18394.97	22080.5	18735.36	21769.4	525.95	300
Clothoid	18735.36	21769.4	18731.73	21729.58	40	
Linear	18731.73	21729.58	18725.27	21672.58	57.36	
Clothoid	18725.27	21672.58	18723.42	21632.69	40	
Circular	18723.42	21632.69	18855.57	21546.83	181.5	-100
Clothoid	18855.57	21546.83	18891.26	21564.72	40	
Linear	18891.26	21564.72	19163.05	21724.43	315.23	
Clothoid	19163.05	21724.43	19197.97	21743.92	40	
Circular	19197.97	21743.92	19557.23	21674.33	393.56	300
Clothoid	19557.23	21674.33	19582.35	21643.21	40	
Linear	19582.35	21643.21	19875.77	21262.69	480.51	

Vertical alignment geometric details

Table 2: Geometric details for vertical alignment

Type	Start			End			Length (m)
	Station	Elevation	Grade	Station	Elevation	Grade	
Linear	0+41.78	371.96	-	0+62.46	367.74	-	20.68
Parabola	0+62.46	367.74	-	5+12.46	413.61	40.80%	450
Linear	5+12.46	413.61	40.80%	5+71.66	437.77	40.80%	59.2
Parabola	5+71.66	437.77	40.80%	9+71.66	406.85	-	400
Linear	9+71.66	406.85	-	11+52.94	304.85	-	181.28
Parabola	11+52.94	304.85	-	14+52.94	262.48	28.01%	300

Linear	14+52.94	262.48	28.01%	15+17.21	280.48	28.01%	64.27
Parabola	15+17.21	280.48	28.01%	17+17.21	273.98	-	200
Linear	17+17.21	273.98	-	20+50.22	159.04	-	333.01
Linear	20+50.22	159.04	-	22+91.26	68.81	-	241.04
Parabola	22+91.26	68.81	-	25+91.26	5.05	-5.07%	300
Linear	25+91.26	5.05	-5.07%	27+23.80	-1.68	-5.07%	132.53

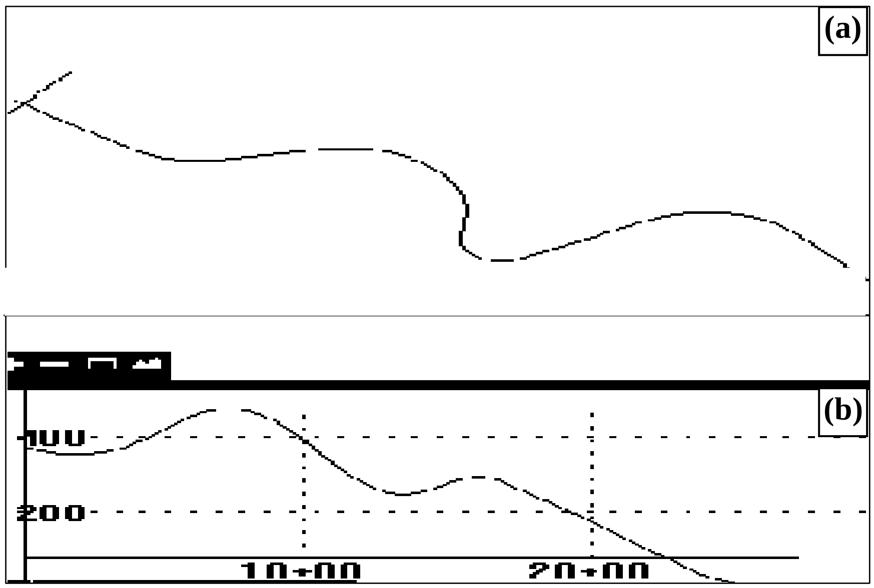

Figure 3: (a). Horizontal alignment for geometric details shown in Table 1 **(b)** Vertical alignment for geometric details shown in Table 2

Alignment created using the proposed method

A total of 14 control points are used to design the alignment using the proposed method. The geometric details of the 3D alignment using the proposed method are shown in Table 3.

Table 3: Geometric details for a 3D alignment

Control Point	Control Point	Type	Start			End		
			x	Y	z	X	y	z
1	2	Linear	18012	20161	371.81	18136	20061	352.35
2	3	Spline	18136	20061	352.35	18407	19899	417.26
3	4	Linear	18407	19899	417.26	18577	19909	464.35
4	5	Spline	18577	19909	464.35	18692	19941	465.63
5	6	Linear	18692	19941	465.63	18811	19957	427.20
6	7	Spline	18811	19957	427.2	18945	19921	350.36
7	8	Linear	18945	19921	350.36	19043	19837	280.57
8	9	Spline	19043	19837	280.57	19103	19650	251.67
9	10	Linear	19103	19650	251.67	19091	19531	281.58

10	11	Spline	19091	19531	281.58	19245	19431	261.89
11	12	Linear	19245	19431	261.89	19547	19608	139.59
12	13	Spline	19547	19608	139.59	19928	19544	11.022
13	14	Linear	19928	19544	11.02	20051	19386	-1.73

Table 4: Spline elements' geometric properties

Start Control Point	End Control Point	c		Comp-onent	$x_t^p = at^3 + bt^2 + ct + d$ Coefficients			
		Start Knot Value	End Knot Value		a	b	c	d
2	3	0	1	x_t^1	-	392.0	124.25	18136
				y_t^1	232.7	-	-	20061
				z_t^1	-102.2	186.5	-	352.35
4	5	0	1	x_t^1	58.32	-	170.33	18577
				y_t^1	-36.31	57.36	10.35	19909
				z_t^1	6.095	-51.90	47.089	464.35
6	7	0	1	x_t^1	-	67.24	118.58	18811
				y_t^1	3.371	-55.16	16.133	19957
				z_t^1	45.45	-83.86	-	427.2
8	9	0	1	x_t^1	-	-3.57	97.825	19043
				y_t^1	170.3	-	-	19837
				z_t^1	17.92	22.97	-69.79	280.57
10	11	0	1	x_t^1	-	183.0	-	19091
				y_t^1	258.5	-	-	19531
				z_t^1	-	3.45	29.906	281.58
12	13	0	1	x_t^1	-	417.7	301.89	19547
				y_t^1	145.9	-	177.09	19608
				z_t^1	122.0	-	-	139.59

From Figure 4, one can see that the alignment sections with straight line elements obtained by the proposed method are matching with some of the existing alignment sections (Figure 3). The other sections, i.e., the spline curved portions of the alignment sections obtained by the proposed method are not matching with the existing curved portion of the alignment. The curved portions are not matching because only two control points are used to fit a spline. If one uses more than two control points, then the curves will match. Thus, the proposed method will effectively replace the two-stage process of alignment design in a single process if sufficient control points were used.

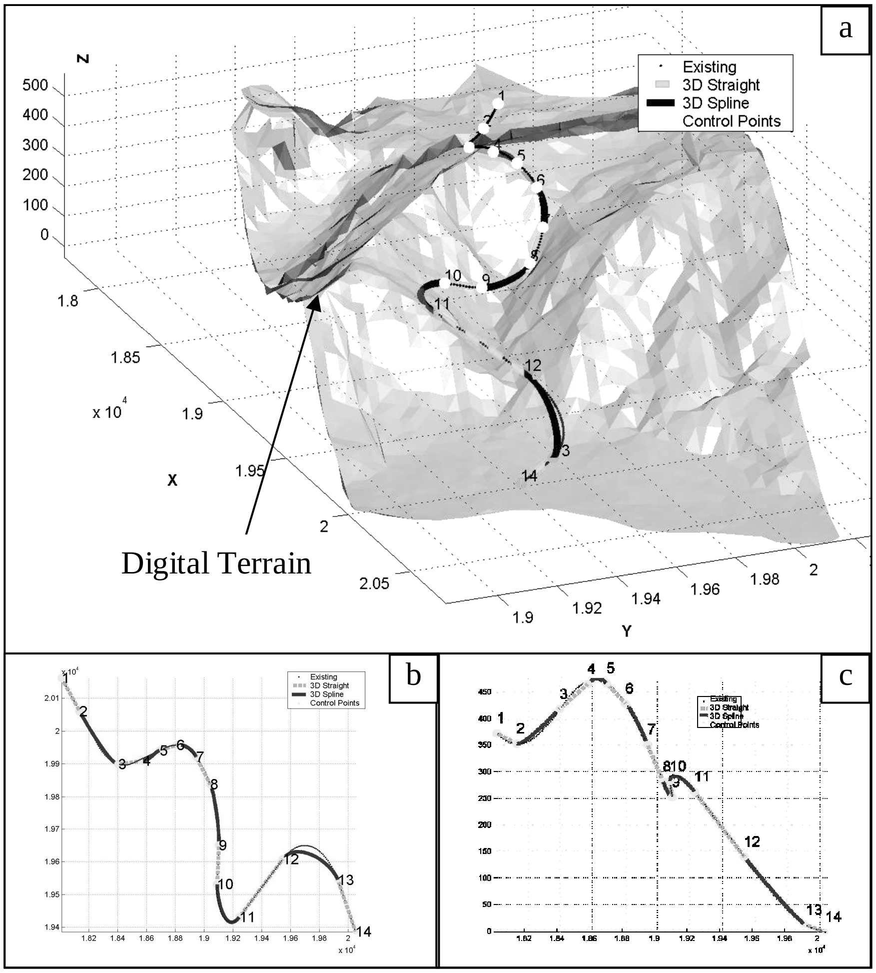

Figure 4: (a) 3D view of alignments obtained using existing and proposed methods. **(b)** Top view of the alignments **(c)** Front view of the alignments.

Investigating the Effects of varying Speeds: Improvement to the Proposed Methodology

Instead of using polynomials, one can use the following parametric equations for *x*, *y*, *z* in terms of the parameter *t*.

$$x = p * \cos(t) \quad (5)$$

$$y = p * \sin(t) \quad (6)$$

$$z = t \quad (7)$$

The principle on selecting a 3D spline (curve) for a particular speed is based on AASHTO's horizontal curve design formula. It can be derived using dynamics of basic physics for motion of a vehicle on curve and is given by:

$$\frac{v^2}{gr} = \frac{0.01e + f}{1 - 0.01ef} \tag{8}$$

where

v = velocity of the vehicle in mph
g = acceleration due to gravity =32.2 f/s^2
e = Roadway Superelevation in percent
f = friction factor between tires of the vehicle and the road
r = radius of curvature of a curve (f)

The above formula states that for a particular superelevation and friction factor, if one needs to increase the design speed of vehicle on the curve, the radius of curvature of the curve has to be increased.

The following terms are introduced for deriving expression of the velocity v and radius of curvature r in terms of Cartesian coordinates:

Let the parameter t denote the time of travel (in seconds) for the vehicle on the alignment. Let $\mathbf{R}(t)$ denote the position vector of vehicle on the alignment and is given by:

Position vector of the alignment, $\mathbf{R}(t) = x(t)\,\mathbf{i} + y(t)\,\mathbf{j} + z(t)\,\mathbf{k}$ (9)

Differentiating the above position vector will result in velocity vector and is written as:

$$\mathbf{v}(t) = \frac{d\mathbf{R}}{dt} = \frac{dx}{dt}\mathbf{i} + \frac{dy}{dt}\mathbf{j} + \frac{dz}{dt}\mathbf{k} \tag{10}$$

The magnitude of the velocity vector is obtained as:

$$v = |\,\mathbf{v}\,| = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2} \tag{11}$$

The direction of the velocity vector is given by:

$$\mathbf{T} = \frac{\mathbf{v}}{|\,\mathbf{v}\,|} \tag{12}$$

Acceleration of the vehicle is given by:

$$\mathbf{a}(t) = \frac{d\mathbf{v}}{dt} = \frac{d^2x}{dt^2}\mathbf{i} + \frac{d^2y}{dt^2}\mathbf{j} + \frac{d^2z}{dt^2}\mathbf{k} \tag{13}$$

Radius of curvature is defined as:

$$r = \frac{|\mathbf{v}|^3}{|\mathbf{v}\mathrm{x}\mathbf{a}|} \tag{14}$$

Differentiating Eqs. (5)-(7) would give the following:

$$\frac{dx}{dt} = -p*\sin(t)\,;\ \frac{dy}{dt} = p*\cos(t)\,;\frac{dz}{dt} = 1 \tag{15}$$

$$\frac{d^2x}{dt^2} = -p*\cos(t)\,;\ \frac{d^2y}{dt^2} = -p*\sin(t)\,;\frac{d^2z}{dt^2} = 0 \tag{16}$$

$$\mathbf{v}(t) = \frac{d\mathbf{r}}{dt} = -p*\sin(t)\,\mathbf{i} + p*\cos(t)\ \mathbf{j} + \mathbf{k} \tag{17}$$

$$\mathbf{a}(t) = \frac{d\mathbf{v}}{dt} = -p*\cos(t)\,\mathbf{i} + -p*\sin(t)\ \mathbf{j} + 0.\,\mathbf{k} \tag{18}$$

$$v = |\mathbf{v}| = \sqrt{(a)^2 + (1)^2} \tag{19}$$

$$\mathbf{v}\mathrm{x}\mathbf{a} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -p*\sin(t) & p*\cos(t) & 1 \\ -p*\cos(t) & -p*\sin(t) & 0 \end{vmatrix} = p*\sin(t)\,\mathbf{i} - p*\cos(t)\ \mathbf{j} + p\mathbf{k} \tag{20}$$

$$r = \frac{|\mathbf{v}|^3}{|\mathbf{v}\mathrm{x}\mathbf{a}|} = \frac{\left(\sqrt{(p)^2 + (1)^2}\right)^3}{p*\sqrt{2}} \tag{21}$$

For a given velocity, superelevation (e), and friction factor one obtain get the radius of curvature (*r*) using Eq. (8). Substituting the computed value of *r* in Eq. (21) and solving the resultant equation would give *p*.

Results and Discussion

Different values of radii obtained for different speeds, superelevation, friction factors and corresponding *p* values are tabulated below in Table 5. The graphical representation of the effect of varying speed on radius of curvature, superelvation, and *p* is shown in Figure 5. The following observations are noted from the inspection of Fig. 5:

1. For higher speeds the value of *p* increases. From the parametric equations (5)-(7) representing the 3D elements, a higher value of *p* implies a wider horizontal alignment (since the z value affecting the vertical alignment is insensitive to P). This seems reasonable since one would expect a wider horizontal curve as the speed increases.
2. As superelevation increases the radius of curvature decreases. This trend is true for any speed.
3. Radius of curvature is higher for higher speeds.
4. Radius of curvature is higher than the *p* value for lower superelevation.

Table 5. Speed, Superelevation, Friction Factor, and *p* Relationships with the Proposed Method

e	*f*	*v*	*r*	*p*
2.0	0.158	60	1344	43.58
2.2	0.158	60	1329	43.34
2.4	0.158	60	1314	43.09
2.6	0.158	60	1299	42.84
2.8	0.158	60	1285	42.61
3.0	0.158	60	1271	42.38
3.2	0.158	60	1257	42.15
3.4	0.158	60	1243	41.91
3.6	0.158	60	1230	41.69
3.8	0.158	60	1217	41.47
4.0	0.158	60	1204	41.25
2.0	0.158	50	933	36.30
2.2	0.158	50	923	36.11
2.4	0.158	50	912	35.89
2.6	0.158	50	902	35.70
2.8	0.158	50	892	35.50
3.0	0.158	50	882	35.30
3.2	0.158	50	873	35.12
3.4	0.158	50	863	34.91
3.6	0.158	50	854	34.73
3.8	0.158	50	845	34.55
4.0	0.158	50	836	34.36
2.0	0.158	40	597	29.03
2.2	0.158	40	591	28.88
2.4	0.158	40	584	28.71
2.6	0.158	40	577	28.54
2.8	0.158	40	571	28.39
3.0	0.158	40	565	28.24
3.2	0.158	40	559	28.09
3.4	0.158	40	553	27.94
3.6	0.158	40	547	27.79
3.8	0.158	40	541	27.63
4.0	0.158	40	535	27.48

Conclusions

A new 3D methodology for design of highways is discussed. The methodology consists of 3D lines (tangents) connected by 3D splines (curves). The 3D lines and splines are defined by *x, y, z* coordinates. The *x, y, z* coordinates of 3D spline is expressed in parametric terms. The relation between speed and radius of curvature from AASHTO are used to get the parametric values for splines. The paper discusses obtaining the parametric values for splines such that the vehicle can safely maneuver the curve for a particular speed.

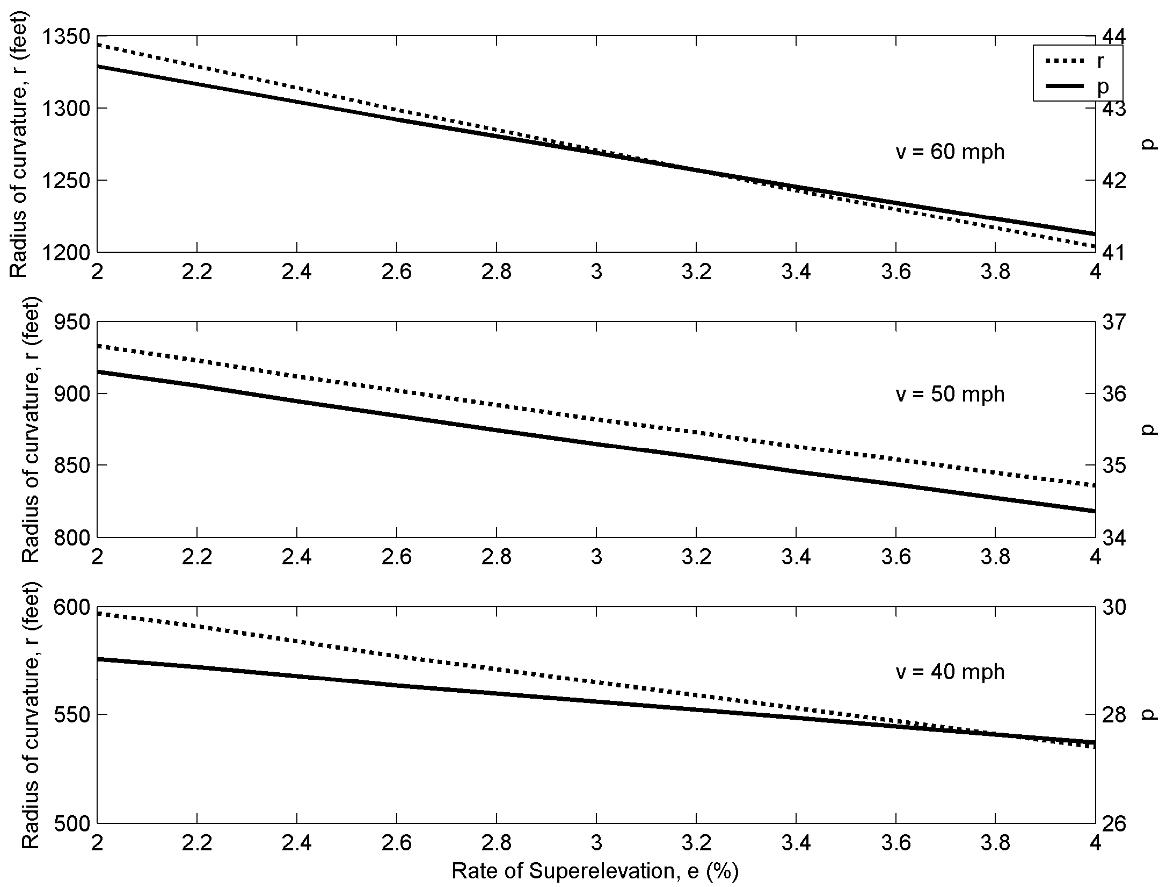

Figure 5. Effects of Varying Speeds on Radius of Curvature, Superelevation, and P.

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